# Factorial Experiments
## 2 x 2 Interactions

Simple Effects:

$a_1b_2 – a_1b_1$
$a_2b_1 – a_1b_1$
$a_2b_2 – a_1b_2$
$a_2b_2 – a_2b_1$

Main Effects:

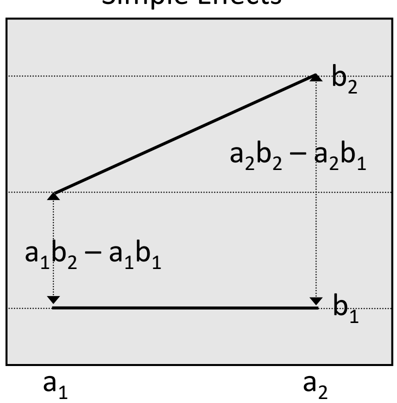
$A = ½[(a_2b_2 – a_1b_2) + (a_2b_1 – a_1b_1)]$
$B = ½[(a_2b_2 – a_2b_1) + (a_1b_2 – a_1b_1)]$

Interaction Effect:

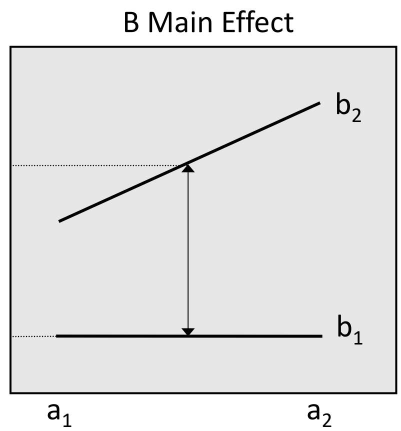
$AB = ½[(a_2b_2 – a_1b_2) - (a_2b_1 – a_1b_1)]$

# Factorial Experiments
## 2 x 2 Interactions

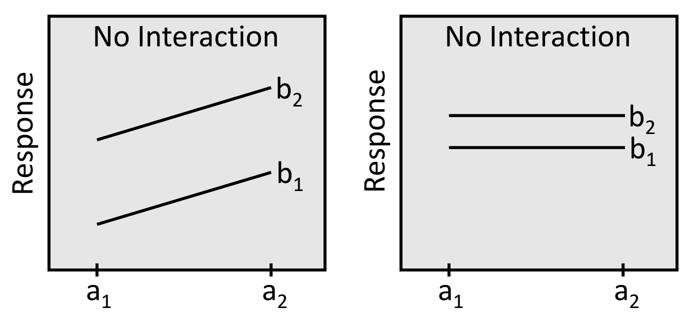
Simple Effects



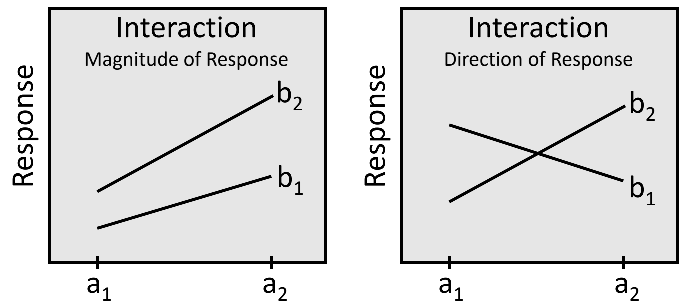
# Factorial Experiments
## 2 x 2 Interactions




# Factorial Experiments
## 2 x 2 Interactions



# Factorial Experiments
## 2 x 2 Interactions




# Factorial Experiments
## Mean Comparisons

<u>Interaction not significant</u>

1. The effect of each treatment does not vary among levels of the other treatment. Simple effects equal across treatment levels.
2. Analysis of means should focus on main effect means.
3. For qualitative treatments where no preplanned comparisons are suggested by treatment structure use an lsd or another multiple comparison procedure.
4. For qualitative treatments where treatment structure suggests preplanned comparisons use linear contrasts to test the hypotheses.
5. For quantitative treatments explore the nature of the response with orthogonal polynomial contrasts and fit the data with an appropriate function.

# Factorial Experiments

## Stockpiled Tall Fescue Example

Treatments

N Source – organic, urea

N Rate – 50, 100, 150, 200 lb/acre

ANOVA

| Source | DF | SS | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Rate | 3 | 18458041 | 6152680 | 170 | <.0001 |
| Source | 1 | 425189.5 | 425189.5 | 11.75 | 0.0022 |
| Rate*Source | 3 | 20475.15 | 6825.05 | 0.19 | 0.9031 |

Because there is no interaction between source and rate of N fertilizer the effect of each main factor should be evaluated separately.

# Factorial Experiments

## Stockpiled Tall Fescue Example

| Source | Mean | N | Group |
|---|---|---|---|
| Urea | 2083.36 | 16 | A |
| Organic | 1852.82 | 16 | B |
| | | | |
| **Rate** | | | |
| 200 | 2902.42 | 8 | A |
| 150 | 2384.80 | 8 | B |
| 100 | 1709.69 | 8 | C |
| 50 | 875.45 | 8 | D |

# Factorial Experiments
## Mean Comparisons

Interaction significant

1. The effect of each treatment varies among levels of the other treatment.
2. Analysis of means should focus on interaction means.
3. When both factors are qualitative:
   - Perform a multiple comparison procedure to identify the best treatment combinations
   - Use contrasts to test specific hypotheses about the treatment means.
4. When one factor is qualitative and the other quantitative examine the functional response of the quantitative factor over each level of the qualitative factor.
5. When both factors are quantitative fit an appropriate response surface.

# Factorial Experiments
## Example: Seeding Date x Rate Study

Model: $Y_{ijk} = \mu + D_i + R_j + DR_{ij} + \varepsilon_{(ij)k}$

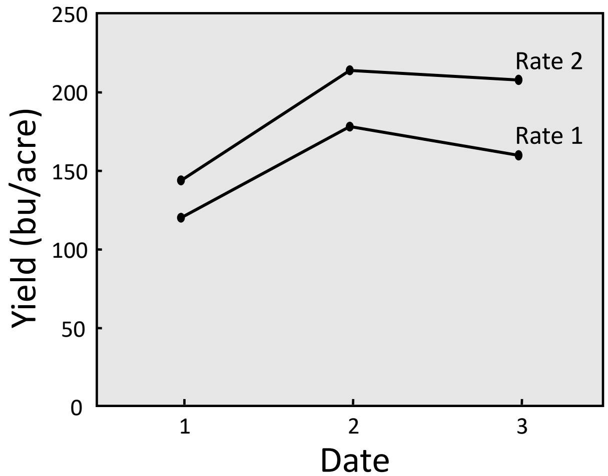
| | | |
|---|---|---|
| Date | $D_i$ | i = 1, 2 ,3 |
| Rate | $R_j$ | j = 1, 2 |

There are 6, 2-way means:

D1R1
D1R2
D2R1
D2R2
D3R1
D3R2

# Factorial Experiments

## Example: Seeding Date x Rate Study




# Factorial Experiments

## Example: Seeding Date x Rate Study

Treatment Means

| Trt | Date | Rate | Mean |
|---|---|---|---|
| 4 | 2 | 2 | 214.0 A |
| 6 | 3 | 2 | 207.9 A |
| 3 | 2 | 1 | 178.2 B |
| 5 | 3 | 1 | 159.9 C |
| 2 | 1 | 2 | 144.0 D |
| 1 | 1 | 1 | 120.2 E |

$$lsd_{.05} = 2.179\sqrt{\frac{2(27.132)}{3}} = 9.26$$

# Factorial Experiments

## Example: Seeding Date x Rate Study

<u>For the comparison</u>: $\mu_{11} - \mu_{12}$

1) Write the hypothesis in terms of model parameters:

   $\mu + D_1 + R_1 + DR_{11} - (\mu + D_1 + R_2 + DR_{12})$

2) Gather like terms:

   $(1 - 1)\mu + (1 - 1)D_1 + (1)R_1 + (-1)R_2 + (1)DR_{11} + (-1)DR_{12}$

3) Coefficients for the contrast are:

   for rate 1 -1
   for date*rate 1 -1 0 0 0 0;

SAS code:

```
contrast 'R1 v 2 in D 1' rate 1 -1 date*rate 1 -1 0 0 0 0;
```

# Factorial Experiments

## Shortcut for Writing Contrasts

| | | Factor B | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | … | b | |
| Factor A | 1 | $c_{11}$ | $c_{12}$ | … | $c_{1b}$ | $c_{1.}$ |
| | 2 | $c_{21}$ | $c_{22}$ | … | $c_{2b}$ | $c_{2.}$ |
| | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | a | $c_{a1}$ | $c_{a2}$ | … | $c_{ab}$ | $c_{a.}$ |
| | | $c_{.1}$ | $c_{.2}$ | … | $c_{.b}$ | $c_{..}$ |

# Factorial Experiments

## Example: Seeding Date x Rate Study

$\mu_{21} - \mu_{22}$

| Date | Rate 1 | Rate 2 | Σ |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 1 | -1 | 0 |
| 3 | 0 | 0 | 0 |
| Σ | 1 | -1 | 0 |

$\mu_{31} - \mu_{32}$

| Date | Rate 1 | Rate 2 | Σ |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 1 | -1 | 0 |
| Σ | 1 | -1 | 0 |

SAS code:

```
contrast  'R1 v 2 in D 2' rate 1 -1 date*rate 0 0 1 -1 0 0;
contrast 'R1 v 2 in D 3' rate 1 -1 date*rate 0 0 0 0 1 -1;
```

# Factorial Experiments

## Example: Seeding Date x Rate Study

Simple Effects:

| Hypothesis | df | Estimate | SS | F |
|---|---|---|---|---|
| $\mu_{11} = \mu_{12}$ | 1 | -23.8 | 849.66 | 31.32** |
| $\mu_{21} = \mu_{22}$ | 1 | -35.8 | 1922.46 | 70.86** |
| $\mu_{31} = \mu_{32}$ | 1 | -48.03 | 3460.80 | 127.56** |

*Conclusion: Rate has an effect at each date.*

# Factorial Experiments

## Example: Seeding Date x Rate Study

Linear Polynomial, Rate 1

| Date | Rate 1 | Rate 2 | Σ |
|---|---|---|---|
| 1 | -1 | 0 | -1 |
| 2 | 0 | 0 | 0 |
| 3 | 1 | 0 | 1 |
| Σ | 0 | 0 | 0 |

Quadratic Polynomial, Rate 1

| Date | Rate 1 | Rate 2 | Σ |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 2 | -2 | 0 | -2 |
| 3 | 1 | 0 | 1 |
| Σ | O | 0 | 0 |

SAS code:

```
contrast 'Date lin, Rate 1' date -1 0 1 date*rate -1 0 0 0 1 0;
contrast 'Date quad, Rate 1' date 1 -2 1 date*rate 1 0 -2 0 1 0;
```

# Factorial Experiments

## Example: Seeding Date x Rate Study

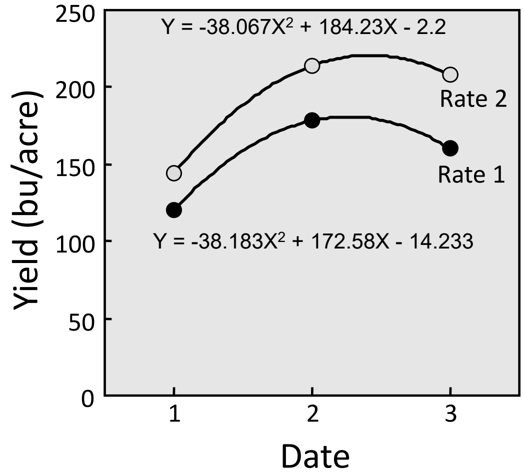
Simple Effects:

| Hypothesis | df | SS | MS | F |
|---|---|---|---|---|
| Rate 1, linear | 1 | 2364.135 | 2364.135 | 87.14** |
| Rate 1, quad | 1 | 2915.934 | 2915.934 | 107.47** |
| Rate 2, linear | 1 | 6131.207 | 6131.207 | 225.98** |
| Rate 2, quad | 1 | 2898.142 | 2898.142 | 106.82** |

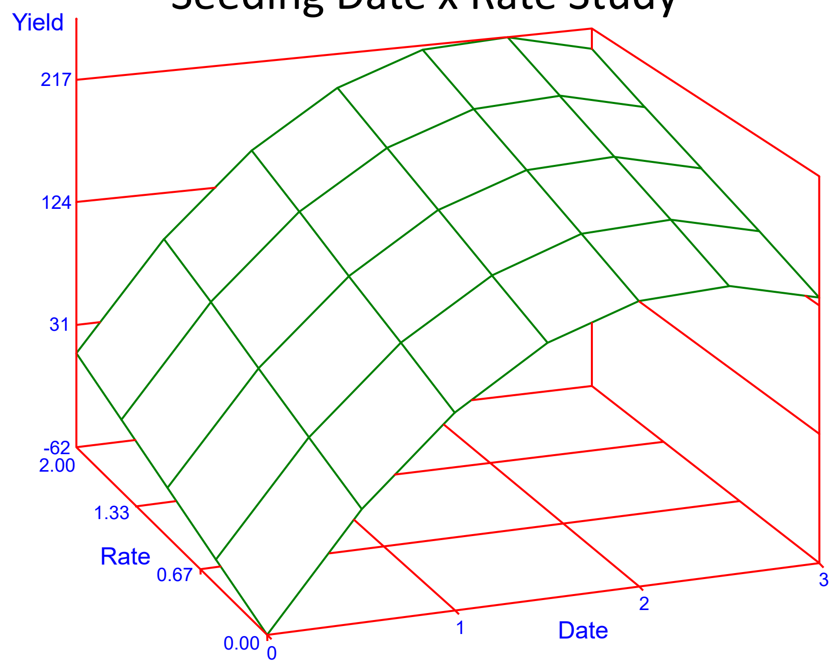
*Conclusion: the response to date was nonlinear for both seeding rates.*

# Factorial Experiments

## Example: Seeding Date x Rate Study




## Seeding Date x Rate Study



## Pseudo Factorial Experiments

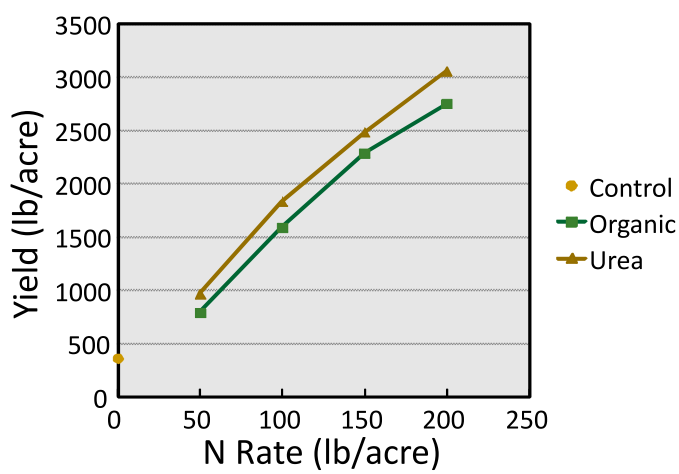
### Redundant Control – Tall Fescue Example

| Source | | Urea | | | | | Organic | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rate | | 0 | 50 | 100 | 150 | 200 | 0 | 50 | 100 | 150 | 200 |
| Rep | 1 | 1 | 5 | 9 | 13 | 17 | 21 | 25 | 29 | 33 | 37 |
| | 2 | 2 | 6 | 10 | 14 | 18 | 22 | 26 | 30 | 34 | 38 |
| | 3 | 3 | 7 | 11 | 15 | 19 | 23 | 27 | 31 | 35 | 39 |
| | 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |

The control rate is the same for each source so you have additional replication of your zero application rate.

## Pseudo Factorial Experiments

### Redundant Control – Tall Fescue Example



# Pseudo Factorial Experiments

## Redundant Control – SAS Code

```
proc glm data=a;
  class treatment;
  model yield = treatment / ss3;
  contrast 'Rate'    treatment 0 1 -1 0 0 0 1 -1 0 0,
                     treatment 0 1 0 -1 0 0 1 0 -1 0,
                     treatment 0 1 0 0 -1 0 1 0 0 -1;
  contrast 'Source' treatment 0 1 1 1 1 0 -1 -1 -1 -1;
  contrast 'R x S'  treatment 0 1 -1 0 0 -1 1 0 0,
                     treatment 0 1 0 -1 0 0 -1 0 1 0,
                     treatment 0 1 0 0 -1 0 -1 0 0 1;
run;
```

# Pseudo Factorial Experiments

## Redundant Control – SAS Results

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Treatment | 9 | 35472736.9 | 3941415.2 | 132.9 | <.0001 |

| Contrast | DF | Contrast SS | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Rate | 3 | 18458040.7 | 6152680.3 | 207.46 | <.0001 |
| Source | 1 | 425189.53 | 425189.53 | 14.34 | 0.0007 |
| R x S | 3 | 20475.15 | 6825.05 | 0.23 | 0.8747 |

# Pseudo Factorial Experiments
## Unbalanced Factorial – Tall Fescue Example

| Source | Control | Urea | | | | Organic | | | |
|---|---|---|---|---|---|---|---|---|---|
| Rate | 0 | 50 | 100 | 150 | 200 | 50 | 100 | 150 | 200 |
| Rep 1 | 1 | 5 | 9 | 13 | 17 | 21 | 25 | 29 | 33 |
| 2 | 2 | 6 | 10 | 14 | 18 | 22 | 26 | 30 | 34 |
| 3 | 3 | 7 | 11 | 15 | 19 | 23 | 27 | 31 | 35 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |

There is only one control rate. The remaining treatments are factorial combinations of rate and source, thus the imbalance in treatments.

# Pseudo Factorial Experiments
## Unbalanced Factorial – SAS Code

```
proc glm data=a;
  class treatment;
  model yield = treatment / ss3;
  contrast 'Rate'   treatment 0 1 -1 0 0 1 -1 0 0,
                    treatment 0 1 0 -1 0 1 0 -1 0,
                    treatment 0 1 0 0 -1 1 0 0 -1;
  contrast 'Source' treatment 0 1 1 1 1 -1 -1 -1 -1;
  contrast 'R x S'  treatment 0 1 -1 0 0 -1 1 0 0,
                    treatment 0 1 0 -1 0 -1 0 1 0,
                    treatment 0 1 0 0 -1 -1 0 0 1;
run;
```